

2. Fractions

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1. What is a Fraction?

A fraction is a number that expresses part of a group.

Fractions are written in the form $\frac{a}{b}$, where a and b are whole numbers, and the number b is not 0.

Why can't "b" be zero?

*Remember: there is no such thing as a number divided by zero
hence there is no such thing as a number over zero.*

The number **a** is called the **numerator**, and the number **b** is called the **denominator**.

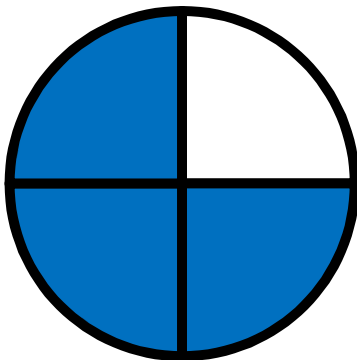
Example:

The following numbers are all fractions

$$\frac{3}{4}, \frac{1}{2}, \frac{4}{99}$$

Example:

The fraction $\frac{3}{4}$ represents the shaded portion of the circle below. There are 4 pieces in the group, and 3 of them are shaded.



2. Improper Fractions

Improper fractions have numerators that are larger than or equal to their denominators.

Examples:

$\frac{11}{4}$, $\frac{5}{5}$, and $\frac{13}{3}$ are improper fractions.

3. Mixed Numbers

Mixed numbers have a whole number part and a fraction part.

Examples:

$2\frac{3}{4}$ and $6\frac{1}{2}$ are mixed numbers.

This means that $2\frac{3}{4}$ is equal to **2** and $\frac{3}{4}$, which is $2 + \frac{3}{4}$.

4. Comparing Fractions

1. To compare fractions with the same denominator, look at their numerators.

The larger fraction is the one with the larger numerator.

Example: $\frac{3}{4}$ is greater than $\frac{1}{4}$ because **3** is greater than **1** (the denominator is the same in both)

2. To compare fractions with the same numerator, look at their denominators.

The larger fraction is the one with the smaller denominator.

Example: $\frac{3}{7}$ is greater than $\frac{3}{8}$ because **7** is smaller than **8** (the numerator is the same in both)

3. To compare fractions with different denominators, take the cross products.

The **first cross-product** is the **product of the first numerator** (on the left) **and the second denominator** (on the right).

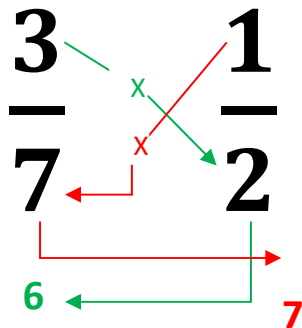
The **second cross-product** is the product of the **second numerator** (on the right) and the **first denominator** (on the left).

Then compare the cross products using the following rules:

- If the cross-products are equal, the fractions are equal. They are called **equivalent fractions**.
- If the first cross product is larger, the first fraction is larger.
- If the second cross product is larger, the second fraction is larger.

Example:

Compare the fractions $\frac{3}{7}$ and $\frac{1}{2}$.



We put the **first cross-product** ($3 \times 2 = 6$) **under the first fraction**.

We put the **second cross-product** ($1 \times 7 = 7$) **under the second fraction**.

Now compare both cross-products, since 7 is bigger than 6 then $\frac{1}{2} > \frac{3}{7}$

Make sure you put the cross-products in their proper places. If you mix them up, you'll get a wrong answer.

Example:

Compare the fractions $\frac{13}{20}$ and $\frac{3}{5}$.

The first cross-product is the product of the first numerator and the second denominator:

$$13 \times 5 = 65.$$

The second cross-product is the product of the second numerator and the first denominator:

$$3 \times 20 = 60.$$

Since the first cross-product is larger, the first fraction is larger.

5. Greatest Common Factor (Sometimes called the Greatest Common Divisor)

Remember how we used to find the factors of a number in chapter one?

To find the greatest common factor of two or more whole numbers, we list the factors of these numbers and then find the **common** factors. The **greatest** of these **common factors** is our greatest common factor (just as the name suggests).

Example: *What is the greatest common factor of 36 and 54?*

List the factors of 36 & 54:

36: 1, 2, 3, 4, 6, 9, 12, **18**, 36

54: 1, 2, 3, 6, 9, **18**, 27, 54

The common factors are: 1, 2, 3, 6, 9, and 18. The greatest common factor is: **18**.

6. Least Common Multiple (Sometimes called the Lowest Common Denominator)

Remember how we used to find the multiples of a number in chapter one?

To find the least common multiple of two or more whole numbers, we list the multiples of these numbers and then find the **common** multiples. The **least** of these **common multiples** is our least common multiple (just as the name suggests).

Example: Find the least common multiple of 12 and 42.

List the multiples of each number, and look for the smallest number that appears in both lists.

12: 12, 24, 36, 48, 60, 72, 84, ...

42: 42, 84, 126, 168, 210, ...

We see that the number 84 is the smallest number that appears in each list.

TRICK:

Always start by finding the multiples of the bigger number. You see, the least common multiple can't be the first three multiples of 12 which are 12, 24, or 36 because they are smaller than 42, which is the first multiple of 42. This means that the first possible common multiple of 12 & 42 can only be 42 or bigger.

7. Multiplying or Dividing the Numerator & Denominator by the Same Number

* For any fraction, multiplying the numerator and denominator by the same nonzero number gives an equivalent fraction. We can convert one fraction to an equivalent fraction by using this method.

Examples:

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} \quad \left(\frac{3}{4} \text{ and } \frac{6}{8} \text{ are equal} \right)$$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \quad \left(\frac{1}{2} \text{ and } \frac{3}{6} \text{ are equal} \right)$$

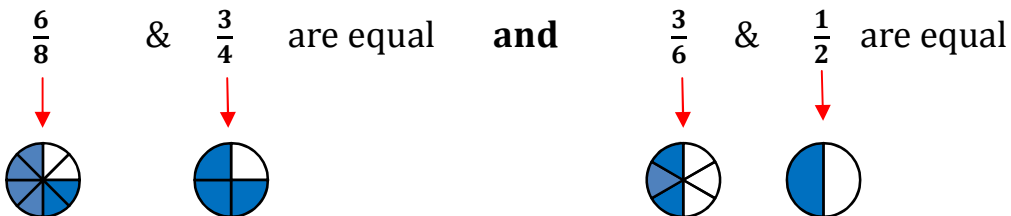
* For any fraction, dividing the numerator and denominator by the same nonzero number gives an equivalent fraction. We can convert one fraction to an equivalent fraction by using this method.

Examples:

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4} \quad \left(\frac{6}{8} \text{ and } \frac{3}{4} \text{ are equal} \right)$$

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2} \quad \left(\frac{3}{6} \text{ and } \frac{1}{2} \text{ are equal} \right)$$

You see, in all the examples above:



8. The Simplest Form of a Fraction (Also called The Lowest Term)

A fraction is in its simplest form or in its lowest terms when the only common factor of its numerator and denominator is 1. To find the simplest form of a fraction:

Divide the numerator and denominator by their greatest common factor.

$$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2} \quad (\text{we divided by 2 because 2 is the greatest common factor of 6 \& 4})$$

$$\frac{7}{49} = \frac{7 \div 7}{49 \div 7} = \frac{1}{7} \quad (\text{we divided by 7 because 7 is the greatest common factor of 7 \& 49})$$

9. Reciprocal

The reciprocal of any number is 1 divided by that number; however, in fractions, 1 divided by a fraction is just the fraction with numerator and denominator switched.

The reciprocal of a fraction is obtained by switching its numerator and denominator.

Example:

The reciprocal of $\frac{23}{4}$ is $\frac{4}{23}$

10. Converting Mixed Numbers to Improper Fractions

To change a mixed number into an improper fraction, multiply the whole number by the denominator then add that product (whole number X denominator) to the numerator of the fraction part. The result is the new numerator. The original denominator does not change.

Examples:

$$2 \frac{3}{4} + = \frac{11}{4}$$

x

11. Converting Improper Fractions to Mixed Numbers

To change an improper fraction into a mixed number, divide the numerator by the denominator. The result is the whole number and the remainder is the numerator of the fractional part. The denominator does not change.

Examples:

$$\frac{11}{4} = 11 \div 4 = 2 \text{ remainder } 3. \text{ Which means } 2 \text{ \& three over four OR } 2 \frac{3}{4}$$

Another way to do this is to write the numerator (11) as two numbers plus each other so that the first number is the biggest possible number divisible by the denominator (4).

Then rewrite the fraction as two fractions plus each other, the first fraction becomes the whole number part of the mixed number and the second fraction becomes the fraction part of the mixed number. Take a look at the example below:

$$\frac{11}{4} = \frac{8+3}{4} = \frac{8}{4} + \frac{3}{4} = 2 + \frac{3}{4} \text{ OR } 2 \frac{3}{4}$$

TRICK:

This is a very useful way of finding remainders, the 3 in the example above.

12. Converting a Fraction to a Decimal

Method 1

Convert to an equivalent fraction whose denominator is a power of 10, such as 10, 100, 1000, 10000, and so on, then write in decimal form by moving the decimal point from the very right to the left a number of times equal to the number of zeros in the denominator.

Examples:

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$$

We multiplied the numerator and denominator by 25 to make the denominator equal to 100 ($25 \times 4 = 100$). Then we moved a decimal point from the right of the "5" two places (because 100 has two zeros) to the left.

Method 2

Divide the numerator by the denominator using regular division or a calculator.

Example:

$$\frac{13}{4} = 13 \div 4 = 3.25$$

13. Adding and Subtracting Fractions

A. If the fractions have the same denominator:

* Their sum is the sum of the numerators over the denominator.

Example:

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

* Their difference is the difference of the numerators over the denominator.

Example:

$$\frac{3}{8} - \frac{2}{8} = \frac{1}{8}$$

Remember: You should never add or subtract the denominators!

B. If the fractions have different denominators:

- 1) First, find the least common denominator.
- 2) Then write equivalent fractions using this denominator.
- 3) Add or subtract the fractions. Reduce if necessary.

Example:

$$\frac{3}{4} - \frac{1}{6} = ?$$

1. The least common denominator (The least common multiple of 4 & 6) is 12.

2. $\frac{9}{12}$ is equivalent to $\frac{3}{4}$ and $\frac{2}{12}$ is equivalent to $\frac{1}{6}$

$$3. \frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

14. Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers, simply convert the mixed numbers into improper fractions, then add or subtract them as fractions.

Example:

$$9\frac{1}{2} + 5\frac{3}{4} =$$

Converting each number to an improper fraction, we get:

$$9\frac{1}{2} = \frac{19}{2} \quad \& \quad 5\frac{3}{4} = \frac{23}{4}$$

We want to calculate $\frac{19}{2} + \frac{23}{4}$. The LCM of 2 and 4 is 4.

$$\text{So, } \frac{19}{2} + \frac{23}{4} = \frac{38}{4} + \frac{23}{4} = \frac{61}{4}$$

15. Multiplying Fractions and Fractions

When two fractions are multiplied, we multiply both numerators and their result becomes the numerator of the answer. Then we multiply both denominators and their result becomes the denominator of the answer.

Example:

$$\frac{3}{7} \times \frac{5}{2} = \frac{3 \times 5}{7 \times 2} = \frac{15}{14}$$

16. Multiplying Fractions and Whole Numbers

To multiply a fraction by a whole number, write the whole number as an improper fraction with a denominator of 1, then multiply as fractions.

Example:

$3 \times \frac{5}{2}$. We can write the number 3 as $\frac{3}{1}$. Now we multiply the fractions.

$$\frac{3}{1} \times \frac{5}{2} = \frac{15}{2}$$

17. Multiplying Mixed Numbers

To multiply mixed numbers, convert them to improper fractions and multiply.

Example:

$$9\frac{1}{2} \times 5\frac{3}{4} =$$

Converting to improper fractions, we get

$$9\frac{1}{2} = \frac{19}{2} \quad \& \quad 5\frac{3}{4} = \frac{23}{4}$$

So the answer is

$$\frac{19}{2} \times \frac{23}{4} = \frac{19 \times 23}{2 \times 4} = \frac{437}{8}$$

18. Dividing Fractions

To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction. Look at the following examples:

$$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$$

$$\frac{2}{3} \div \frac{7}{4} = \frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$$

Notes:

1. If you are dividing a whole number by a fraction or a fraction by a whole number, simply write the whole number as a fraction with a denominator of 1 then solve as explained above. Confused? Well, here's an example:

Example:

$$\frac{2}{3} \div 7 = \frac{2}{3} \div \frac{7}{1} = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$$

2. If you are asked to divide mixed numbers, just convert them to improper fractions then proceed like explained above.

Example:

$$9\frac{1}{2} \div 5\frac{3}{4} = ?$$

First convert them into improper fractions, so the question becomes:

$$\frac{19}{2} \div \frac{23}{4} =$$

$$\frac{19}{2} \div \frac{23}{4} = \frac{19}{2} \times \frac{4}{23} = \frac{76}{46}$$

Remember that multiplying or dividing the numerator by the same number gives us an equivalent form of the fraction. So, to make this fraction simpler we can divide the numerator and denominator by 2. This will give us

$$\frac{76}{46} = \frac{76 \div 2}{46 \div 2} = \frac{38}{23}$$

19. Simplifying Complex Fractions

A complex fraction is a fraction whose numerator or denominator is also a fraction or mixed number. To solve these types of fractions, simply rewrite the question as a division question.

Example:

$$\frac{\frac{2}{3}}{\frac{7}{5}} = \frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$